MATHEMATICS

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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XI (PQRS)

TRIGONOMETRIC EQUATIONS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. An equation containing trigonometric functions of unknown angles is known as a trigonometric equation.
2. A solution of a trigonometric equation is the value of the unknown angle that satisfies the equation.
3. Following are the general solutions of trigonometric equations in standard frosm :

Trigonometric equation	**General solution**
(i) $\sin\theta = 0$	$\theta = n\pi,\ n \in Z$
(ii) $\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2},\ n \in Z$
(iii) $\tan\theta = 0$	$\theta = n\pi,\ n \in Z$
(iv) $\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n \alpha,\ n \in Z$
(v) $\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha,\ n \in Z$
(vi) $\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha,\ n \in Z$
(vii) $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\theta$	$\theta = n\pi \pm \alpha,\ n \in Z$

4. The euqation $a\cos\theta + b\sin\theta = c$ is solvable for $|c| \le \sqrt{a^2+b^2}$

EXERCISE-1

1. Prove that the general solution of $\tan\theta = 0$ is $\theta = n\pi,\ n \in N$.
2. Prove that the general solution of $\cos\theta = 0$ is $\theta = (2n+1)\frac{\pi}{2},\ n \in Z$.
3. Prove that the general solution of $\cos\theta = 0$ is $\theta = (2n+1)\frac{\pi}{2},\ n \in Z$.
4. Find the general solutions of the following equations :

   (i) $\tan 2\theta = 0$ (ii) $\tan\frac{\theta}{2}$ (iii) $\tan\frac{36}{4} = 0$

5. Prove that the general solution of $\sin\theta = \sin\alpha$ is given by : $\theta = n\pi + (-1)^n \alpha,\ n \in Z$
6. Solve the equation : $\sin\theta = \sin 3\theta + \sin 5\theta = 0$
7. Solve the equation : $\sin m\theta + \sin n\theta = 0$.
8. Solve the following equations :
   (i) $\sin 2\theta + \cos\theta = 0$
   (ii) $\sin 3\theta + \cos 2\theta = 0$
   (iii) $\sin 2\theta + \sin 4\theta + \sin 6\theta = 0$

9. Solve the following equations :

(i) $2\cos^2\theta + 3\sin\theta = 0$

(ii) $\cot^2\theta + \frac{3}{\sin\theta} + 3 = 0.$

(iii) $2\tan\theta - \cot\theta = -1$

(iv) $4\cos\theta - 3\sec\theta = \tan\theta$

(v) $\tan^2\theta + \left(1-\sqrt{3}\right)\tan\theta - \sqrt{3} = 0$

(vi) $\sec^2 2x = 1 - \tan 2x$

10. Solve the following equations :

(i) $\tan\theta + \tan 2\theta + \tan\theta\tan 2\theta = 1$

(ii) $\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta\tan 2\theta\tan 3\theta$

(iii) $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

(iv) $\tan\theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$

11. Prove that :

(i) $\sin^2\theta = \sin^2\alpha \Rightarrow n\pi \pm \alpha,\ n \in Z$

(ii) $\cos^2\theta = \cos^2\alpha \Rightarrow n\pi \pm \alpha,\ n \in Z$

(iii) $\tan^2\theta = \tan^2\alpha \Rightarrow n\pi \pm \alpha,\ n \in Z$

12. Solve : $7\cos^2\theta + 3\sin^2\theta = 4$

13. Solve : $4\sin x\sin 2x\sin 4x = \sin 3x$

14. Solve : $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$

15. Solve the general solutions of the following equations :

(i) $\sin 2\theta = \frac{\sqrt{3}}{2}$

(ii) $\cos 3\theta = \frac{1}{2}$

(iii) $\sin 9\theta = \sin\theta$

(iv) $\sin 2\theta = \cos 3\theta$

(v) $\tan\theta + \cot 2\theta = 0$

(vi) $\tan 3\theta = \cot\theta$

(vii) $\tan 2\theta\ \tan\theta = 1$

(viii) $\tan m\theta + \cot n\theta = 0$

(ix) $\tan p\theta = \cot q\theta$

(x) $\sin 2\theta + \cos\theta = 0$

(xi) $\sin\theta = \tan\theta$ d

16. Solve the following equations :

(i) $\sin^2\theta - \cos\theta = \frac{1}{4}$

(ii) $2\cos^2\theta - 5\cos\theta + 2 = 0$

(iii) $2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

(iv) $4\sin^2\theta - 8\cos\theta + 1 = 0$

(v) $\tan^2 x + \left(1-\sqrt{3}\right)\tan x - \sqrt{3}$

(vi) $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos - 3\sin^2\theta = 0$

(vii) $\cos 4\theta = \cos 2\theta$

17. Solve the following equations :

(i) $\tan\theta + \tan 2\theta + \tan 3\theta = 0$

(ii) $\tan\theta + \tan 2\theta = \tan 3\theta$

(iii) $\tan 3\theta + \tan\theta = 2\tan 2\theta$

EXERCISE-2

Answer each of the following questions in one word or one sentence of as per exact requirement of the questions :

1. Write the number of solutions of the equation $\tan x + \sec x = 2 \cos x$ in the interval $[0, 2\pi]$.
2. Write the number of solutions of the equation $4 \sin x - 3 \cos x = 7$.
3. Write the number of points of intersection of the curves $2y = 1$ and $y = \cos x$, $0 \le x \le 2\pi$.
4. Write the values of x in $[0, \pi]$ for which $\sin 2x$, $\frac{1}{2}$ and $\cos 2x$ in A.P.
5. Write the number of points of intersection of the curves $2y = -1$ and $y = \text{cosec } x$.
10. Write the number of values of θ in $[0, 2\pi]$ that satisfy the equation $\sin^2\theta - \cos\theta = \frac{1}{4}$.

EXERCISE-3

Mark the correct alternative in each of the following

1. If $\cos\theta + \sqrt{3}\sin\theta = 2$, then $\theta =$

   (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

2. If a is any real number, the number of roots of $\cot x - \tan x = a$ in the first quadrant is (are).

   (a) 2 (b) 0 (c) 1 (d) none of these

3. The general solution of the equation $7\cos^2\theta + 3\sin^2\theta = 4$ is

   (a) $\theta = 2n\pi \pm \frac{\pi}{6}, n \in Z$ (b) $\theta = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

   (c) $\theta = n\pi \pm \frac{\pi}{3}, n \in Z$ (d) none of these

4. A solution of the equation $\cos^2\theta + \sin\theta + 1 = 0$, lies in the interval

   (a) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (c) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (d) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

5. The smallest positive angle which satisfies the equation : $2\sin^2\theta + \sqrt{3}\cos\theta + 1 = 0$ is

   (a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

6. If $4\sin^2\theta = 1$, then values of θ are

   (a) $2n\pi \pm \frac{3\pi}{3}, n \in Z$ (b) $n\pi \pm \frac{\pi}{3}, n \in Z$

   (c) $n\pi \pm \frac{3\pi}{6}, n \in Z$ (d) $2n\pi \pm \frac{3\pi}{6}, n \in Z$

7. A value of θ satisfying $\cos\theta + \sqrt{3}\sin\theta = 2$ is

(a) $\frac{5\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

8. The number of values of θ in $[0, 2\pi]$ that satisfy the equation $\sin^2\theta - \cos\theta = \frac{1}{4}$

(a) 1 (b) 2 (c) 3 (d) 4

9. If $e^{\sin x} - e^{-\sin x} - 4 = 0$, thenn x =

(a) 0

(b) $\sin^{-1}\left\{\log_e\left(2-\sqrt{5}\right)\right\}$

(c) 1

(d) none of these

10. General solution of $\tan 5\theta = \cot 2\theta$ is

(a) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}, n \in Z$

(b) $\theta = \frac{n\pi}{7} + \frac{\pi}{3}, n \in Z$

(c) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}, n \in Z$

(d) $\theta = \frac{n\pi}{7} - \frac{\pi}{14}, n \in Z$